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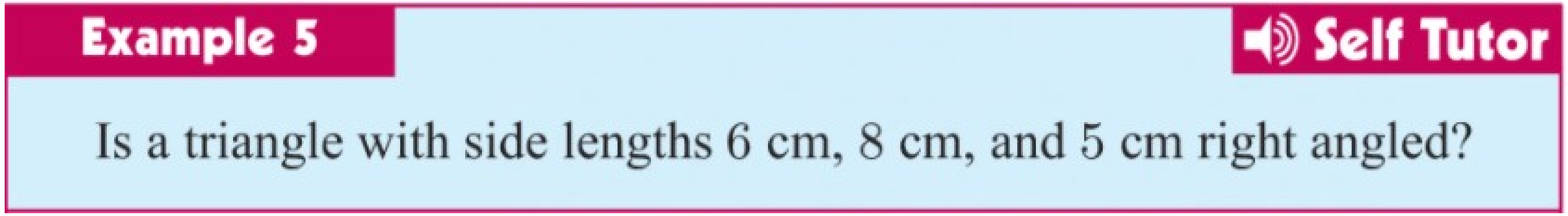
If a triangle has sides of length a, b, and c units where $a^2 + b^2 = c^2$, then the triangle is right angled.

Example 5 **Self Tutor**

Is a triangle with side lengths 6 cm, 8 cm, and 5 cm right angled?



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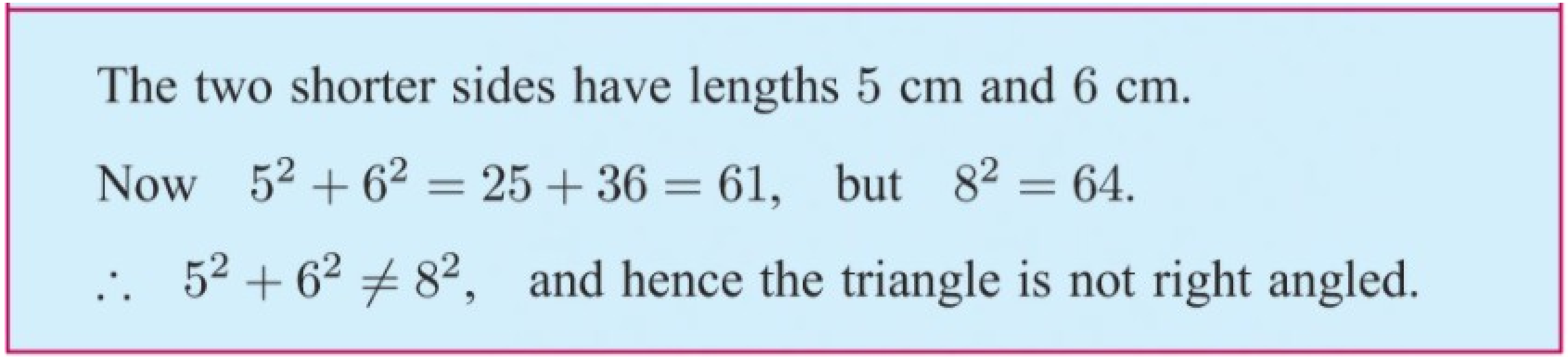

The two shorter sides have lengths 5 cm and 6 cm.

Now $5^2 + 6^2 = 25 + 36 = 61$, but $8^2 = 64$.

$\therefore$ $5^2 + 6^2 \neq 8^2$, and hence the triangle is not right angled.

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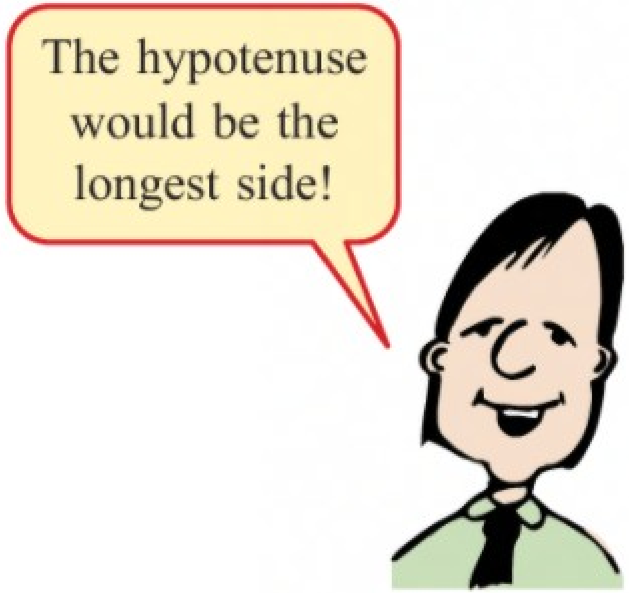
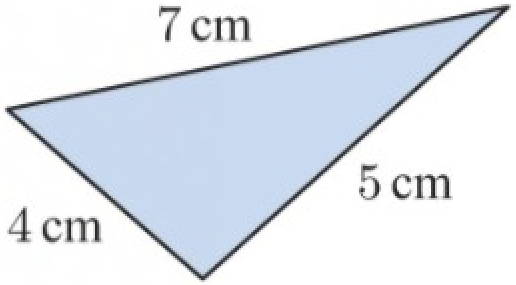

The hypotenuse would be the longest side!

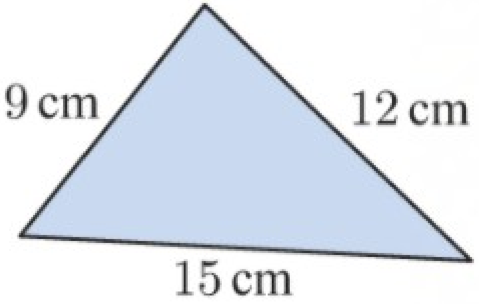
EXERCISE 5B

1 The following figures are not drawn to scale. Which of the triangles are right angled?

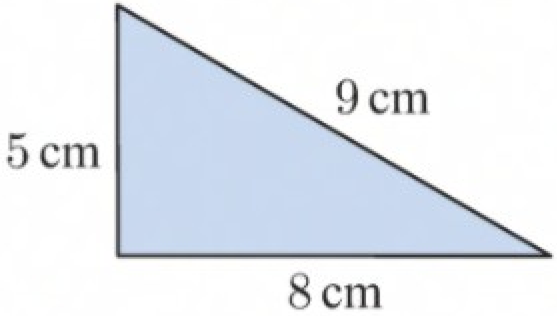
a



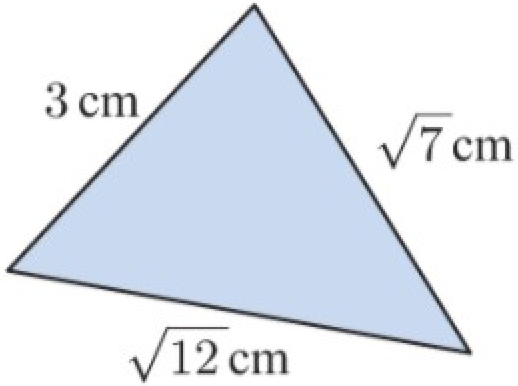
b



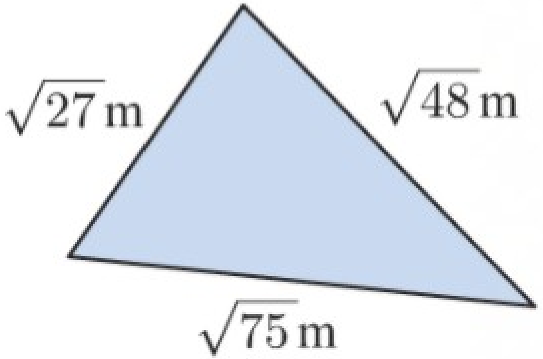
c



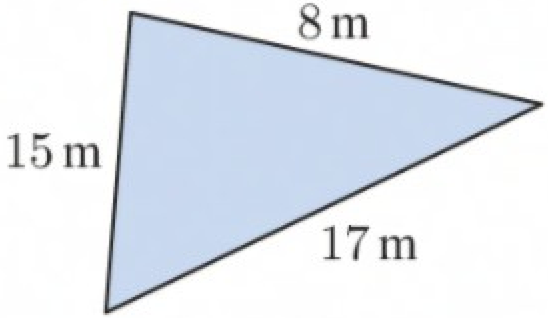
d



e

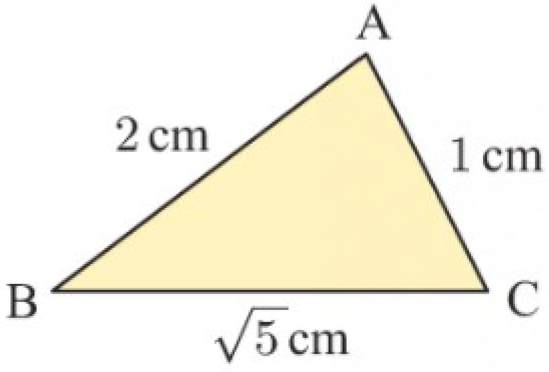


f

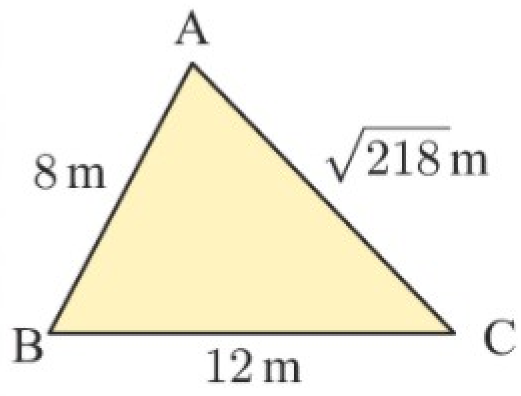


2 The following triangles are not drawn to scale. If any of them is right angled, identify the right angle.

a



b



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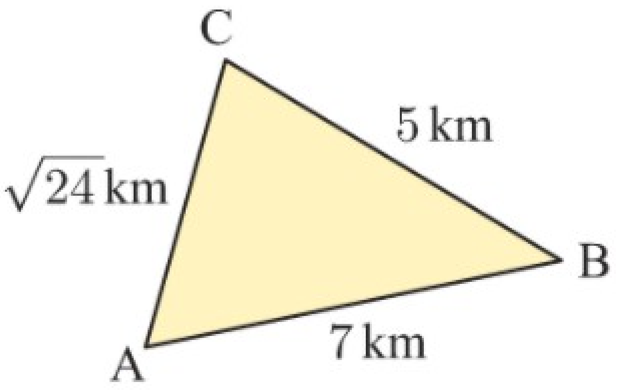

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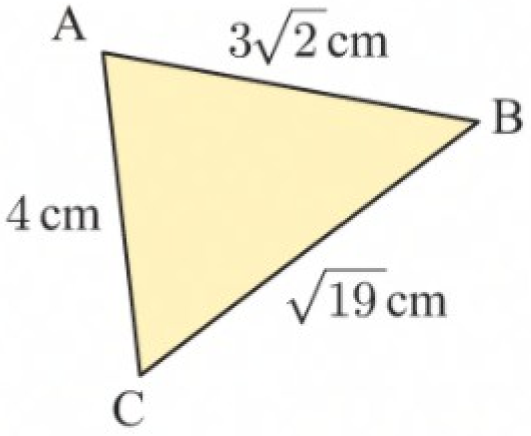

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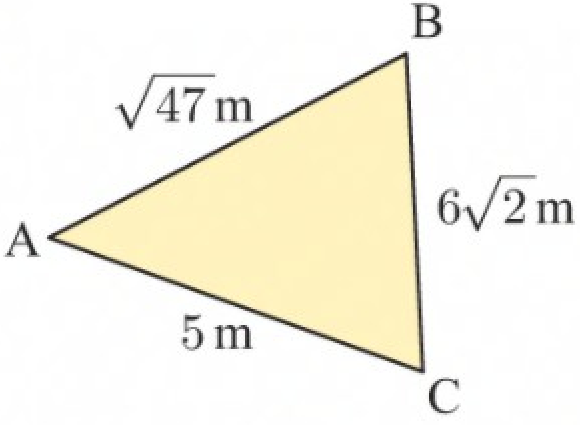

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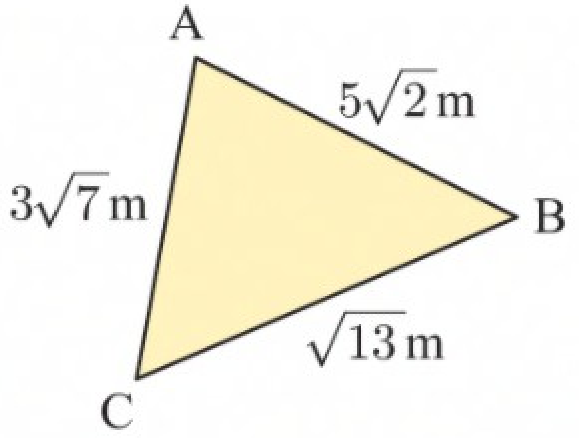
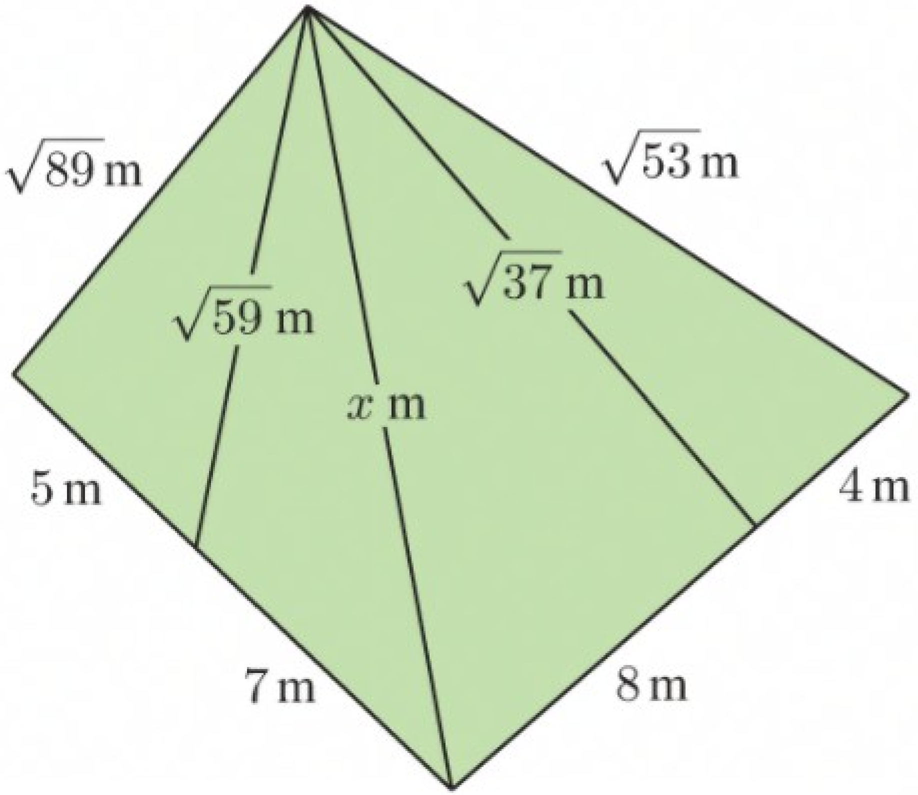


3 Find x:



C

PYTHAGOREAN TRIPLES

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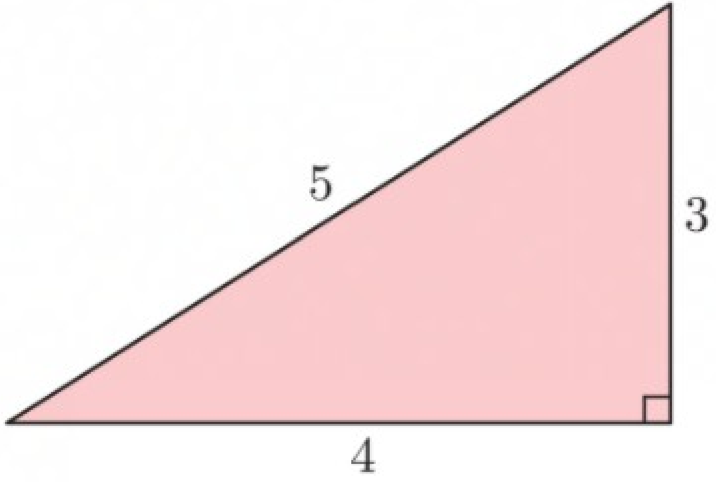

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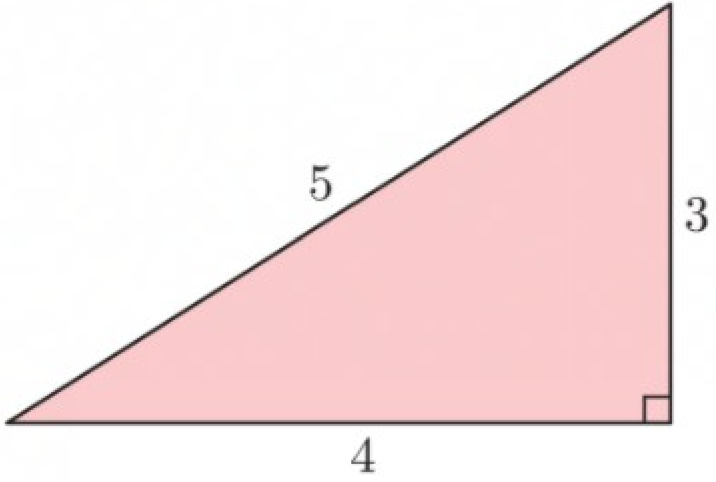

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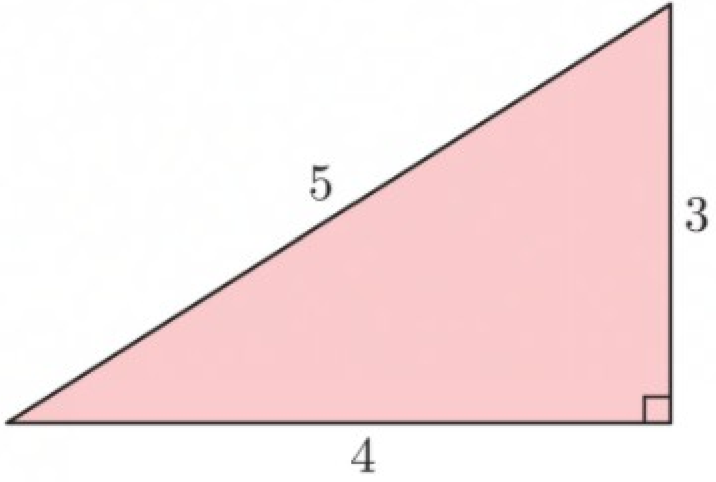

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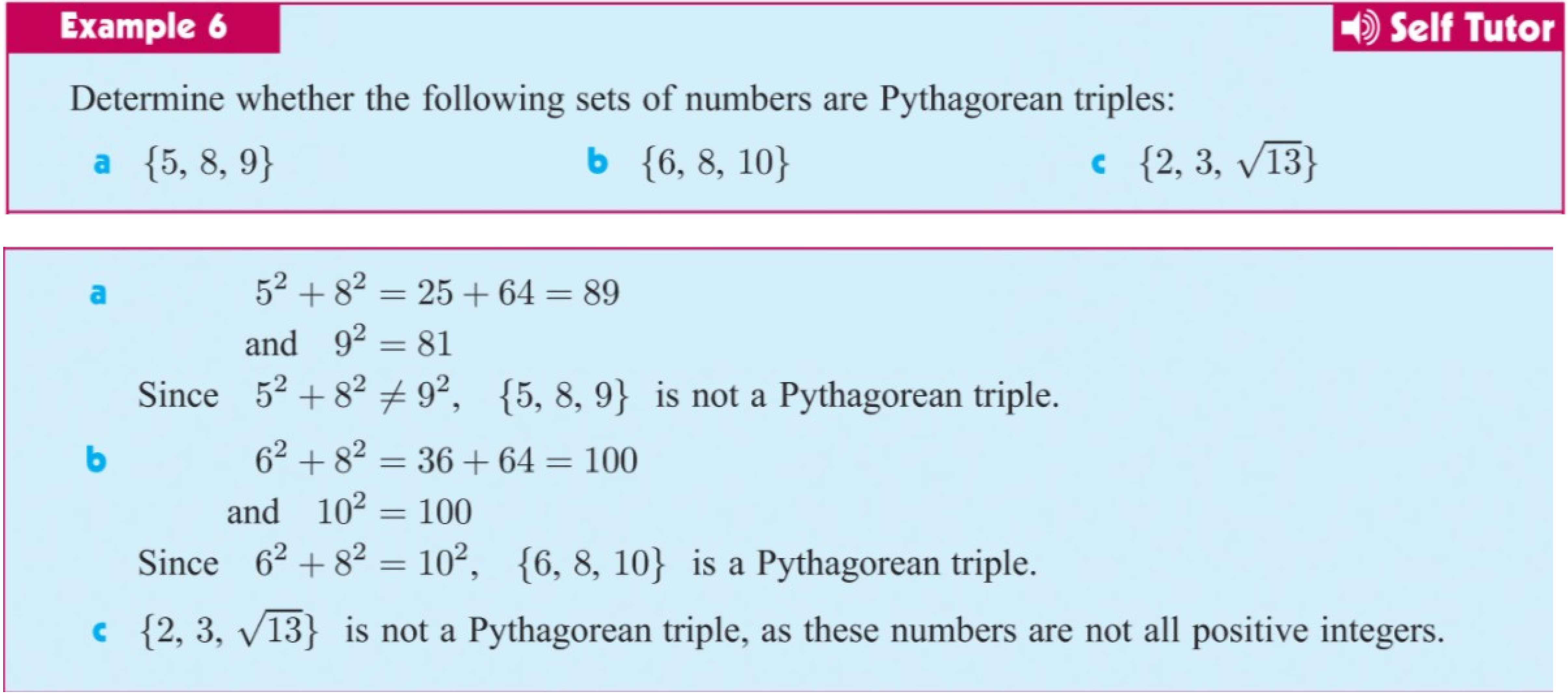
Pythagorean triples correspond to right angled triangles with sides of integer length.

Example 6

Self Tutor

Determine whether the following sets of numbers are Pythagorean triples:

a $\{5, 8, 9\}$ **b** $\{6, 8, 10\}$ **c** $\{2, 3, \sqrt{13}\}$



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a $\{5, 8, 9\}$ **b** $\{6, 8, 10\}$ **c** $\{2, 3, \sqrt{13}\}$

a $5^2 + 8^2 = 25 + 64 = 89$

and $9^2 = 81$

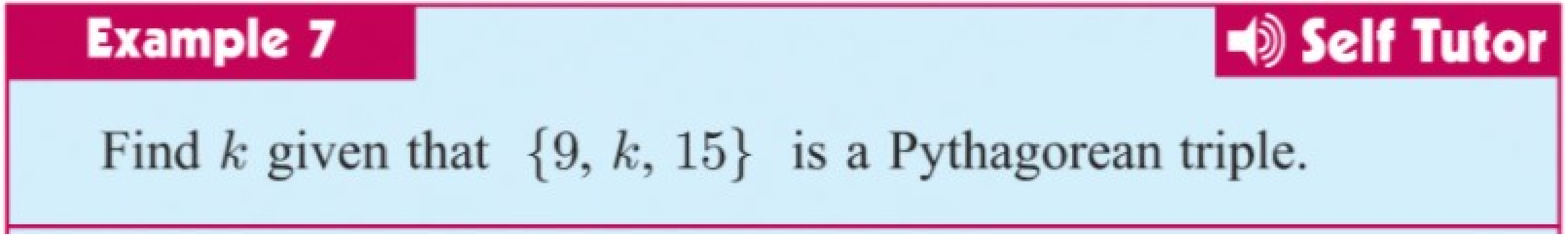
Since $5^2 + 8^2 \neq 9^2$, $\{5, 8, 9\}$ is not a Pythagorean triple.

b $6^2 + 8^2 = 36 + 64 = 100$

and $10^2 = 100$

Since $6^2 + 8^2 = 10^2$, $\{6, 8, 10\}$ is a Pythagorean triple.

c $\{2, 3, \sqrt{13}\}$ is not a Pythagorean triple, as these numbers are not all positive integers.



Example 7 **Self Tutor**

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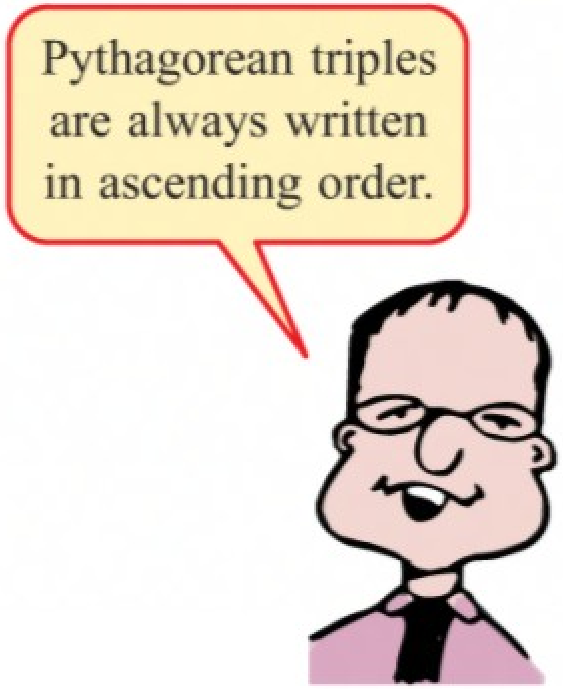
Pythagorean triples are always written in ascending order.

EXERCISE 5C

1 Determine whether the following are Pythagorean triples:

a $\{15, 20, 25\}$ **b** $\{5, 6, 7\}$ **c** $\{14, 48, 50\}$

d $\{1, 6, \sqrt{37}\}$ **e** $\{20, 48, 52\}$ **f** $\{-15, 8, 17\}$

2 Find k given that the following are Pythagorean triples:

a $\{12, 16, k\}$ **b** $\{k, 24, 26\}$ **c** $\{14, k, 50\}$

d $\{8, k, k+2\}$ **e** $\{20, k, k+8\}$ **f** $\{k, 60, k+50\}$

3 **a** Given that $\{a, b, c\}$ is a Pythagorean triple and k is a positive integer, show that $\{ka, kb, kc\}$ is also a Pythagorean triple.

b Use the multiples $k = 2, 3, 4, \text{ and } 5$ to construct new Pythagorean triples from these Pythagorean triples:

i $\{3, 4, 5\}$ **ii** $\{5, 12, 13\}$

4 **a** Given that $\{a, b, c\}$ and $\{d, e, f\}$ are Pythagorean triples, show that $\{be - ad, bd + ae, cf\}$ is also a Pythagorean triple.

b Given that $\{3, 4, 5\}$ and $\{8, 15, 17\}$ are Pythagorean triples, use **a** to construct a new Pythagorean triple. Use technology to check your answer.

5 **a** For each of these values of n, show that $\{2n+1,\ 2n^2+2n,\ 2n^2+2n+1\}$ is a Pythagorean triple:

i $n = 1$ **ii** $n = 2$ **iii** $n = 3$ **iv** $n = 4$

b Prove that $\{2n+1,\ 2n^2+2n,\ 2n^2+2n+1\}$ is a Pythagorean triple for all $n \in \mathbb{Z}^+$.

Hint: Let $a = 2n+1$, $b = 2n^2+2n$, and $c = 2n^2+2n+1$, then simplify $c^2 - b^2 = (2n^2+2n+1)^2 - (2n^2+2n)^2$ using the *difference of two squares* factorisation.

EXERCISE 5B

1 **a** No $\{$as $4^2+5^2=41$ and $7^2=49\}$

b Yes $\{$as $9^2+12^2=225$ and $15^2=225\}$

c No $\{$as $5^2+8^2=89$ and $9^2=81\}$

d No $\{$as $3^2+(\sqrt{7})^2=16$ and $(\sqrt{12})^2=12\}$

e Yes $\{$as $(\sqrt{27})^2+(\sqrt{48})^2=75$ and $(\sqrt{75})^2=75\}$

f Yes $\{$as $8^2+15^2=289$ and $17^2=289\}$

2 **a** As $(\sqrt{5})^2=1^2+2^2$, it is right angled at A.

b As $8^2+12^2=208\neq(\sqrt{218})^2$, it is not right angled.

c As $7^2=49=(\sqrt{24})^2+5^2$, it is right angled at C.

d As $4^2+(3\sqrt{2})^2\neq(\sqrt{19})^2$, it is not right angled.

e $5^2+(\sqrt{47})^2=72=(6\sqrt{2})^2$

$\therefore$ it is right angled at A.

f $(\sqrt{13})^2+(5\sqrt{2})^2=13+50=63$ and $(3\sqrt{7})^2=63$

$\therefore$ it is right angled at B.

3 $x=\sqrt{101}\approx 10.0$

Hint: Show right-most triangle to be right angled first.

EXERCISE 5C

1 **a** yes **b** no **c** yes
d no {numbers must be integers} **e** yes
f no {numbers must be positive}

2 **a** $k = 20$ **b** $k = 10$ **c** $k = 48$ **d** $k = 15$
e $k = 21$ **f** $k = 11$

3 **a** As $\{a, b, c\}$ is a Pythagorean triple, $a^2 + b^2 = c^2$

Hint: $(ka)^2 + (kb)^2 = k^2a^2 + k^2b^2 = k^2(a^2 + b^2)$

b **i** $\{6, 8, 10\}$, $\{9, 12, 15\}$, $\{12, 16, 20\}$, $\{15, 20, 25\}$
ii $\{10, 24, 26\}$, $\{15, 36, 39\}$, $\{20, 48, 52\}$, $\{25, 60, 65\}$

4 **a** Show $(be - ad)^2 + (bd + ae)^2 = (cf)^2$
b $\{36, 77, 85\}$

5 **a** **i** $\{3, 4, 5\}$, $3^2 + 4^2 = 5^2$
ii $\{5, 12, 13\}$, $5^2 + 12^2 = 13^2$
iii $\{7, 24, 25\}$, $7^2 + 24^2 = 25^2$
iv $\{9, 40, 41\}$, $9^2 + 40^2 = 41^2$